First Idea For A Ranking Based Semantic Using System Z

Kenneth Skiba

Institute for Web Science and Technologies, University of Koblenz-Landau, Germany,
email: kennethskiba@uni-koblenz.de

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Abstract

We discuss ranking arguments from an Dung-style argumentation framework with the help of conditional logics. Using an intuitive translation for an argumentation framework to generate conditionals, we can apply nonmonotonic inference systems to generate a ranking on these conditionals. With this ranking we construct a ranking for our arguments.

1 Introduction

Abstract argumentation has become a popular topic in artifical intelligence, mainly in the area of Knowledge Representation and Reasoning. Especially in this area, ranking the solutions is often discussed. With a ranking over arguments, we can determine "good" arguments or relationships between arguments. For example, if we accept an argument a then we are not allow to also accept argument b, that is attacked by a, because there is a contradiction between a and b.

There are already ideas for ranking arguments, named ranking-based semantics [Amgoud and Ben-Naim, 2013], like a ranking with respect to a categoriser function [Pu et al., 2014] or based on a two-person zero-sum strategic game [Matt and Toni, 2008] and many more. [Delobelle, 2017] summarizes the state-of-the-art models for ranking arguments. We want to use conditional logics for our ranking model.

The rest of this work is organized as follows: In Section 2 all necessary preliminaries will be stated. Then we discus our ranking idea in Section 3 and with Section 4 we conclude this paper.

2 Method

In the following, we want to briefly recall some general preliminaries on conditional logic and argumentation frameworks.

2.1 Abstract Argumentation Frameworks

In this work we use the idea of *argumentation frameworks* first introduced in [Dung, 1995]. An *argumentation framework AF* is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a finite set of arguments and $\mathcal{R}$ is a set of attacks between arguments with $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. An argument a is said to *attack* b if $(a, b) \in \mathcal{R}$. We call an argument a *acceptable with respect to a set* $S \subseteq \mathcal{A}$ if for each attacker $b \in \mathcal{A}$ of this argument a with $(b, a) \in \mathcal{R}$, there is an argument $c \in S$ which attacks b, i.e., $(c, b) \in \mathcal{R}$; we then say that a is *defended by* c. An argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ can be illustrated by a directed graph with vertex set $\mathcal{A}$ and edge set $\mathcal{R}$.

Example 1. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(a, b), (b, c), (c, d), (d, c)\}$ be an argumentation framework. The corresponding graph is shown in Figure 1. Argument b is not acceptable with respect to any set S of arguments, as b is not defended

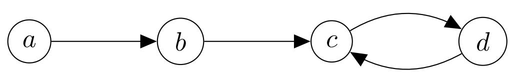

Figure 1: Argumentation framework from Example 1

against a's attack. On the other hand, c is acceptable with respect to $S = \{a, c\}$, as a defends c against b's attack and c defends itself against d's attack.

Up to this point the arguments can only have the two statuses of accepted or not accepted[1], but we want to have a more fine-graded comparison between arguments. For this we use the idea of rankings-based semantics based on [Amgoud and Ben-Naim, 2013] and [Delobelle, 2017].

Definition 2. (Ranking-based semantics)

A *ranking-based semantics* σ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{\sigma}_{AF}$ on $\mathcal{A}$ where $\succeq^{\sigma}_{AF}$ is a preorder on $\mathcal{A}$. $a \succeq^{\sigma}_{AF} b$ means that a is at least as acceptable as b. With $a \simeq^{\sigma}_{AF} b$ we describe that a and b are equally acceptable. Finally we say a is strictly more acceptable than b, when $a \succ^{\sigma}_{AF} b$. We denote by $\sigma(AF)$ the ranking on $\mathcal{A}$ returned by σ.

2.2 Conditional Logics

A *possible world* for a formula is a function, which evaluates a atom a in this formula with $\top$ or $\bot$. We say w satisfies an atom a if $w(a) = \top$, written $w \vdash a$. We denote the set of all interpretations as $\Omega(A)$. An interpretation w *satisfies* (written as $w \vdash a$) an atom $a \in A$, if and only if $w(a) = \top$. We will abbreviate an interpretation w with its *complete conjuction*, i.e., if $a_1, \ldots, a_n \in A$ are the atoms that are assigned $\top$ by w and $a_{n+1}, \ldots, a_m \in A$ are the ones assigned with $\bot$, w will be identify $a_1 \ldots a_n \overline{a_{n+1}} \ldots \overline{a_m}$.

For conditional logics we use the approach from [De Finetti, 2017], who considers conditionals as *generalized indicator functions* for possible worlds resp. propositional interpretations w:

$$((\varphi|\phi))(w) = \begin{cases} 1 : w \vdash \phi \wedge \varphi \ (verifies) \\ 0 : w \vdash \phi \wedge \neg\varphi \ (falsifies) \\ u : w \vdash \neg\phi \ (\textit{not applicable}) \end{cases} \tag{1}$$

where u stand for *unknown*. Informal speaking a world w *verifies* a conditional $(\varphi|\phi)$ iff it satisfies both antecedent and conclusion $((\varphi|\phi)(w) = 1)$; it *falsifies* iff is satisfies the antecedence but not the conclusion $((\varphi|\phi)(w) = 0)$; otherwise the conditional is *not applicable* $((\varphi|\phi)(w) = u)$. A conditional $(\varphi|\phi)$ is satisfied by w if it does not falsify it. We can interpret this conditionals as a three-valued model and therefore be compatible with the probabilistic interpretation of conditionals as conditional probabilities. So a conditional $(\varphi|\phi)$ can be accepted as plausible if its verification $\varphi \wedge \phi$ is more plausible than its falsification $\phi \wedge \neg\varphi$. With a ranking function, also called *ordinal conditional function (OCF)*, $\kappa : \Omega \rightarrow \mathbb{N} \cup \{\infty\}$ we can express the degree of plausibility of possible worlds $\kappa(\phi) := min\{\kappa(w)|w \vdash \phi\}$. With the help of OCFs κ we can express the acceptance of conditionals and nonmonotonic inferences, so $(\varphi|\phi)$ is accepted by κ iff $\kappa(\phi \wedge \varphi) < \kappa(\phi \wedge \varphi)$. With $Bel(\kappa) = \{\phi | \forall w \in \kappa^{-1}(0) : w \vdash \phi\}$ we denote the most plausible worlds.

Now we want to define a inference relation, namely System Z $\mid\!\sim_{\mathrm{Z}}$ [Goldszmidt and Pearl, 1996]. With this model we can rank the possible worlds by there satisfiability. So a world, which does not falsifies a condition has a better rank then a world, which falsifies a condition.

Definition 3. (System Z)

$(\varphi|\phi)$ is tolerated by a finite set of conditionals Δ if there is a possible world w with $(\phi|\varphi)(w) = 1$ and $(\phi'|\varphi')(w) \neq 0$ for all $(\phi'|\varphi') \in \Delta$. The *Z-partition* $(\Delta_0, \ldots, \Delta_n)$ of Δ is defined as:

- $\Delta_0 = \{\delta \in \Delta | \Delta \ \textit{tolerates} \ \delta\}$
- $\Delta_1, \ldots, \Delta_n$ is the Z-partition of $\Delta \setminus \Delta_0$

For $\delta \in \Delta$: $Z_\Delta(\delta) = i$ iff $\delta \in \Delta_i$ and $\Delta_1, \ldots, \Delta_n$ is the Z-partitioning of Δ.

We define a ranking function $\kappa^Z_\Delta : \Omega \rightarrow \mathbb{N} \cup \{\infty\}$ as $\kappa^Z_\Delta(w) = max\{Z(\delta)|\delta(w) = 0, \delta \in \Delta\} + 1$, with

[1]Using labeling-based semantics we can generate a three-valued model [Wu et al., 2010].

$max\ \emptyset = -1$. Finally $\Delta \mathrel{|\!\sim}_Z \phi$ if and only if $\phi \in Bel(\kappa^Z_\Delta)$, with $Bel(\kappa^Z_\Delta) = \{\phi | \forall w \in \kappa^{-1}(0) : w \vdash \phi\}$.

Example 4. Let $\Delta = \{(a|\neg b), (b|\neg a), (c|\neg b \wedge \neg a \wedge \neg d), (d|\top), (c|\neg d)\}$. For this set of conditionals, $\Delta = \Delta_0 \cup \Delta_1$ with $\Delta_0 = \{(a|\neg b), (b|\neg a), (c|\neg b \wedge \neg a \wedge \neg d)\}$ and $\Delta_1 = \{(\neg a \wedge \neg b|\ d)\}$ therefore we have the values from Table 1. So we can derive $(\kappa^Z_{\Delta_0})^{-1}(0) = \{abcd, ab\bar{c}d, a\bar{b}cd, a\bar{b}\bar{c}d, \bar{a}bcd, \bar{a}b\bar{c}d\}$ and $(\kappa^Z_{\Delta_1})^{-1}(0) = \emptyset$.

3 Discussion

In this work we want to extend the work of [Heyninck et al., 2020] and [Kern-Isberner and Thimm, 2018] to not only combine abstract argumentation and conditional logics, but also present ideas to rank arguments using this combination.

First we need a translation from an argumentation framework to conditional logics. It is clear, that for an argument to be acceptable every attacker has to be not acceptable. With this idea we can construct the conditional logic knowledge base. Let AF be an argumentation framework and $\theta : \mathcal{A} \to \mathcal{C}_\mathcal{A}$, where $\mathcal{C}_\mathcal{A}$ is the set of conditional knowledge bases over the propositional language generated by $\mathcal{A}$.

$$\theta(AF) = \{(a|B) | (a \in \mathcal{A}), B = \neg b_1 \wedge \neg b_2 \wedge \ldots \wedge \neg b_n,\ where\ (b_i, a) \in \mathcal{R}\} \tag{2}$$

We can use inference systems like system Z [Goldszmidt and Pearl, 1996] on these conditional knowledge bases to generate a ranking over these worlds. Based on this ranking we want to rank the arguments. This idea is still work in progress. A first idea is to count the number of occurrences of a positive literal in the set of worlds $(\kappa^Z_\Delta)^{-1}(0)$ and then rank the corresponding arguments based on this number. So if an argument a has a higher count then an argument b, we say $a \succeq b$.

Definition 5. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ be an argumentation framework translated with help of $\theta(AF)$ and a inference system to the set of worlds κ^Z_Δ. Define

$$Ccs^\theta_{\kappa^Z_\Delta(\omega)}(a) = |\{w \in (\kappa^Z_\Delta)^{-1}(0) | w \vdash a\}| \tag{3}$$

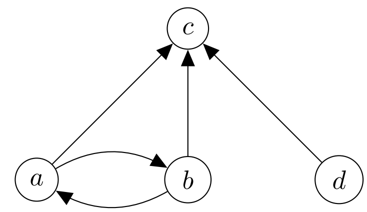

Figure 2: Argumentation framework from Example 7

We can then use this counting function for our ranking-based semantics.

Definition 6. (Conditional-counting-based semantic)

The *Conditional-counting-based semantic (Ccbs)* associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a ranking $\succeq^{Ccbs}_{AF}$ on $\mathcal{A}$ such that $\forall a, b \in \mathcal{A}$ with respect to a transition θ and a ranking function $\kappa^Z_\Delta(\omega)$.

$$a \succeq^{Ccbs}_{AF} b \text{ if and only if } Ccs^\theta_{\kappa^Z_\Delta(\omega)}(a) \geq Ccs^\theta_{\kappa^Z_\Delta(\omega)}(b)$$

Example 7. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(a, b), (b, a), (a, c), (b, c), (d, c)\}$ be an argumentation framework. The corresponding graph can be found in Figure 2. Using Equation 2 we obtain $\Delta = \{(a|\neg b), (b|\neg a), (c|\neg b \wedge \neg a \wedge \neg d), (d|\top)\}$. With $\Delta = \Delta_0$ we have $(\kappa^Z_\Delta)^{-1}(0) = \{abcd, ab\bar{c}d, a\bar{b}cd, a\bar{b}\bar{c}d, \bar{a}bcd, \bar{a}b\bar{c}d\}$. Now we can count the number of occurrences of each argument. So $Ccs^\theta_{\kappa^Z_\Delta(\omega)}(a) = 4$, $Ccs^\theta_{\kappa^Z_\Delta(\omega)}(b) = 4$, $Ccs^\theta_{\kappa^Z_\Delta(\omega)}(c) = 3$ and $Ccs^\theta_{\kappa^Z_\Delta(\omega)}(d) = 6$. This results in $d \succeq^{Ccbs} a \simeq^{Ccbs} b \succeq^{Ccbs} c$.

A problem with our translation is that it is not Z-adequate [Heyninck et al., 2020], therefore it would be better to use another translation. But a ranking using this translation should still produce meaningful results. For some ideas of better translations we recommend [Heyninck et al., 2020]. Instead of system Z we could also use c-representations [Kern-Isberner, 2001].

To evaluate a ranking semantics we need general properties. We want to look at two simple ones and evaluate our semantics with them. Namely *Void*

Table 1: Values for Example 4

ω	$Z((a\|\neg b))$	$Z((b\|\neg a))$	$Z((c\|\neg b \wedge \neg a \wedge \neg d))$	$Z((d\|\top))$	$Z((\neg a \wedge \neg b\|d))$
$abcd$	u	u	u	1	0
$abc\overline{d}$	u	u	u	0	u
$ab\overline{c}d$	u	u	u	1	0
$ab\overline{c}\overline{d}$	u	u	u	0	u
$a\overline{b}cd$	1	u	u	1	0
$a\overline{b}c\overline{d}$	1	u	u	0	u
$a\overline{b}\overline{c}d$	1	u	u	1	0
$a\overline{b}\overline{c}\overline{d}$	1	u	u	0	u
$\overline{a}bcd$	u	1	u	1	0
$\overline{a}bc\overline{d}$	u	1	u	0	u
$\overline{a}b\overline{c}d$	u	1	u	1	0
$\overline{a}b\overline{c}\overline{d}$	u	1	u	0	u
$\overline{a}\overline{b}cd$	0	0	u	1	1
$\overline{a}\overline{b}c\overline{d}$	0	0	1	0	u
$\overline{a}\overline{b}\overline{c}d$	0	0	u	1	1
$\overline{a}\overline{b}\overline{c}\overline{d}$	0	0	0	0	u

Precedence [Matt and Toni, 2008, Amgoud and Ben-Naim, 2013] and *Self-Contradiction* [Matt and Toni, 2008]. The idea of *Void Precedence* states that a non-attacked argument should be strictly more acceptable than an attacked argument.

Definition 8. (Void Precedence)
A ranking-based semantics σ satisfies *Void Precedence* if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall a, b \in \mathcal{A}$, if $\forall c \in \mathcal{A}$ $(c, a) \notin \mathcal{R}$ and $\exists d \in \mathcal{A}$ with $(d, b) \in \mathcal{R}$, then $a \succ^{\sigma}_{AF} b$.

On the contrary, a self-attacking argument should always be ranked worse than any other argument, because these arguments are contradicting themselves. This is handled with the property *Self-Contradiction*.

Definition 9. (Self-Contradiction)
A ranking-based semantics σ satisfies *Self-Contradiction* if and only if for any $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ and $\forall a, b\mathcal{A}$, if $(a, a) \notin \mathcal{R}$ and $(b, b) \in \mathcal{R}$ then $a \succ^{\sigma}_{AF} b$.

Proposition 10. Ccbs *does not satisfy Void Precedence nor Self-Contradiction.*

Proof. To prove this we look at the following example. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A} = \{a, b\}$ and $\mathcal{R} = \{(a, a)\}$ be an argumentation framework. Using Equation 2 we obtain $\Delta = \{(a|\neg a), (b|\top)\}$. With $\Delta = \Delta_0$ we have $(\kappa^Z_\Delta)^{-1}(0) = \{ab\}$ and $Ccs^{\theta}_{\kappa^Z_\Delta(\omega)}(a) = 1$, $Ccs^{\theta}_{\kappa^Z_\Delta(\omega)}(b) = 1$. This results in $a \simeq^{Ccbs} b$. Therefore a is not strictly less acceptable then b. So *Ccbs* does not satisfy Void Precedence nor Self-Contradiction. ❑

Hence this semantics has a few shortcomings, but an extension, which solves the problem of self-attacking arguments, should yield a reasonable ranking-based semantic. A full analysis for such a semantics will be presented in a follow-up work.

4 Conclusion

In this work we have presented a first idea to rank arguments with conditional logics. For this we first looked at a simple transition from an argumentation framework to conditional logic and applied an inference relation. Using a simple counting idea results in a ranking over arguments.

Although this semantics does not satisfy two desired properties, we have a established simple connection between ranking arguments and conditional logic. In the future we can improve this idea and hopefully present a ranking-based semantic, which satisfies a good number of properties presented in [Delobelle, 2017]. Another future work approach is to look at other frameworks like ADFs presented in [Brewka et al., 2013], which uses an acceptance function for every argument. This could prove to be helpful in finding a ranking with conditional logic.

[Kern-Isberner and Simari, 2011] used a similar idea to rank arguments from a *Defeasible Logic Programming* (DeLP), a system, which combines logics programming with defeasible argumentation. They used system Z to identify "good" arguments.

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