Towards a Ranking-Based Semantics using System Z

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Abstract. We discuss ranking arguments from an Dung-style argumentation framework with the help of conditional logics. Using an intuitive translation for an argumentation framework to generate conditionals, we can apply nonmonotonic inference systems to generate a ranking on these conditionals. With this ranking we construct a ranking for our arguments.

1 INTRODUCTION

Formal argumentation [2] describes a family of approaches to modeling rational decision-making through the representation of arguments and their relationships. A particular important representative approach is that of abstract argumentation [6], which focuses on the representation of arguments and a conflict relation between arguments through modeling this setting as a directed graph. Here, arguments are identified by vertices and an *attack* from one argument to another is represented as a directed edge. This simple model already provides an interesting object of study, see [3] for an overview. Reasoning is usually performed in abstract argumentation by considering *extensions*, i. e., sets of arguments that are jointly acceptable given some formal account of "acceptability". Therefore, this classical approach differentiates between "acceptable" arguments and "rejected" arguments.

However, ranking-based semantics [1] provide a finer-grained assessment of arguments. For this idea there is already a line of work like a ranking with respect to a categoriser function [11] or based on a two-person zero-sum strategic game [10] and many more. [5] summarizes the state-of-the-art models for ranking arguments.

We want to make some first steps towards the use of conditional logic and the System Z inference mechanism [7] to define rankings between arguments. Conditional logic is a general non-monotonic representation formalism that focuses on default rule of the form "if A then B" and there exist some interesting relationships between this formalism and that of formal argumentation [8, 9]. We make use of these relationships here for the purpose of defining a novel ranking-based semantics for abstract argumentation.

2 ABSTRACT ARGUMENTATION FRAMEWORKS

In this work, we use *argumentation frameworks* first introduced in [6]. An *argumentation framework* AF is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a finite set of arguments and $\mathcal{R}$ is a set of attacks between arguments with $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. An argument a is said to *attack* b if $(a, b) \in \mathcal{R}$. We call an argument a *acceptable with respect to a set* $S \subseteq \mathcal{A}$ if for each attacker $b \in \mathcal{A}$ of this argument a with $(b, a) \in \mathcal{R}$, there is an argument $c \in S$ which attacks b, i. e., $(c, b) \in \mathcal{R}$; we then say that a is *defended by* c. An argumentation framework $\langle \mathcal{A}, \mathcal{R} \rangle$ can be illustrated by a directed graph with vertex set $\mathcal{A}$ and edge set $\mathcal{R}$.

Example 1. Let $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(a, b), (b, c), (c, d), (d, c)\}$ be an argumentation framework. Argument b is not acceptable with respect to any set S of arguments, as b is not defended against a's attack. On the other hand, c is acceptable with respect to $S = \{a, c\}$, as a defends c against b's attack and c defends itself against d's attack.

Up to this point the arguments can only have the two statuses of accepted or not accepted[2], but we want to have a more fine-graded comparison between arguments. For this we use the idea of ranking-based semantics [1, 5].

Definition 2 (Ranking-based semantics)**.** A *ranking-based semantics* σ associates to any argumentation framework $AF = \langle \mathcal{A}, \mathcal{R} \rangle$ a preorder $\succeq^{\sigma}_{AF}$ on $\mathcal{A}$. $a \succeq^{\sigma}_{AF} b$ means that a is at least as acceptable as b. With $a \simeq^{\sigma}_{AF} b$ we describe that a and b are equally acceptable, i. e., $a \succeq^{\sigma}_{AF} b$ and $b \succeq^{\sigma}_{AF} a$. Finally we say a is strictly more acceptable than b, denoted by $a \succ^{\sigma}_{AF} b$, if $a \succeq^{\sigma}_{AF} b$ and not $b \succeq^{\sigma}_{AF} a$. We denote by $\sigma(AF)$ the ranking on $\mathcal{A}$ returned by σ.

3 CONDITIONAL LOGICS

We use a set of atoms A and connectives $\wedge$ (and), $\vee$ (or), and $\neg$ (negation) to generate the *propositional language* $\mathcal{L}(A)$. w is an *interpretation* (or *possible world*) for $\mathcal{L}(A)$ when $w : A \to \{\text{TRUE}, \text{FALSE}\}$. We denote the set of all interpretations as $\Omega(A)$. An interpretation w *satisfies* an atom $a \in A$ ($w \vdash a$), if and only if $w(a) = \text{TRUE}$. The relation $\vdash$ is extended to arbitrary formulas in the usual way. We will abbreviate an interpretation w with its *complete conjunction*, i. e., if $a_1, \ldots, a_n \in A$ are the atoms that are assigned TRUE by w and $a_{n+1}, \ldots, a_m \in A$ are the ones assigned with FALSE, w will be identified with $a_1 \ldots a_n \overline{a_{n+1}} \ldots \overline{a_m}$. For $\Phi \subseteq \mathcal{L}\{A\}$ we define $w \vdash \Phi$ if and only if $w \vdash \phi$ for every $\phi \in \Phi$. With $Mod(X) = \{w \in \Omega(A) | w \vdash X\}$ we define the set of models for a set of formulas X. A *conditional* is a structure of the form $(\varphi|\psi)$ and represents a rule "If ψ than (usually) ϕ".

We can consider conditionals as *generalized indicator functions* [4] for possible worlds w as follows:

$$((\varphi|\phi))(w) = \begin{cases} 1 : w \vdash \phi \wedge \varphi \\ 0 : w \vdash \phi \wedge \neg\varphi \\ u : w \vdash \neg\phi \end{cases} \tag{1}$$

where u stands for *unknown*. Informally speaking, a world w *verifies* a conditional $(\varphi|\phi)$ iff it satisfies both antecedent and conclusion

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[2] However, using labeling-based semantics we can generate a three-valued model [14].

$((\varphi|\phi)(w) = 1)$; it *falsifies* iff is satisfies the antecedence but not the conclusion $((\varphi|\phi)(w) = 0)$; otherwise the conditional is *not applicable* $((\varphi|\phi)(w) = u)$. A conditional $(\varphi|\phi)$ is satisfied by w if it does not falsify it.

Semantics are given to sets of conditionals via ranking functions [7,13]. With a ranking function, also called *ordinal conditional function (OCF)*, $\kappa : \Omega(A) \rightarrow \mathbb{N} \cup \{\infty\}$ we can express the degree of plausibility of possible worlds $\kappa(\phi) := min\{\kappa(w)|w \vdash \phi\}$. With the help of OCFs κ we can express the acceptance of conditionals and nonmonotonic inferences, so $(\varphi|\phi)$ is accepted by κ iff $\kappa(\phi \wedge \varphi) < \kappa(\phi \wedge \varphi)$. With $Bel(\kappa) = \{\phi|\forall w \in \kappa^{-1}(0) : w \vdash \phi\}$ we denote the most plausible worlds.

As there are an infinite number of ranking functions that accept a given set of conditionals, we consider System Z [7] as an inference relation, which yields us a uniquely defined ranking function for reasoning.

Definition 3 (System Z)**.** $(\varphi|\phi)$ is tolerated by a finite set of conditionals Δ if there is a possible world w with $(\phi|\varphi)(w) = 1$ and $(\phi'|\varphi')(w) \neq 0$ for all $(\phi'|\varphi') \in \Delta$. The *Z-partition* $(\Delta_0, \ldots, \Delta_n)$ of Δ is defined as:

- $\Delta_0 = \{\delta \in \Delta | \Delta \text{ tolerates } \delta\}$
- $\Delta_1, \ldots, \Delta_n$ is the Z-partition of $\Delta \setminus \Delta_0$

For $\delta \in \Delta$: $Z_\Delta(\delta) = i$ iff $\delta \in \Delta_i$ and $\Delta_1, \ldots, \Delta_n$ is the Z-partitioning of Δ.

We define a ranking function $\kappa^Z_\Delta : \Omega \rightarrow \mathbb{N} \cup \{\infty\}$ as $\kappa^Z_\Delta(w) = max\{Z(\delta)|\delta(w) = 0, \delta \in \Delta\} + 1$, with $max\ \emptyset = -1$. Finally $\Delta \mid\!\sim_Z \phi$ if and only if $\phi \in Bel(\kappa^Z_\Delta)$.

Example 4. Let $\Delta = \{(a|\neg b), (b|\neg a), (c|\neg b \wedge \neg a \wedge \neg d), (d|\top), (c|\neg d)\}$. For this set of conditionals, $\Delta = \Delta_0 \cup \Delta_1$ with $\Delta_0 = \{(a|\neg b), (b|\neg a), (c|\neg b \wedge \neg a \wedge \neg d)\}$ and $\Delta_1 = \{(\neg a \wedge \neg b|\, d)\}$ therefore we have the values from the following table.

ω	$Z((a\|\neg b))$	$Z((b\|\neg a))$	$Z((c\|\neg b \wedge \neg a \wedge \neg d))$	$Z((d\|\top))$	$Z((\neg a \wedge \neg b\|d))$
$abcd$	u	u	u	1	0
$abc\bar{d}$	u	u	u	0	u
$ab\bar{c}d$	u	u	u	1	0
$ab\bar{c}\bar{d}$	u	u	u	0	u
$a\bar{b}cd$	1	u	u	1	0
$a\bar{b}c\bar{d}$	1	u	u	0	u
$a\bar{b}\bar{c}d$	1	u	u	1	0
$a\bar{b}\bar{c}\bar{d}$	1	u	u	0	u
$\bar{a}bcd$	u	1	u	1	0
$\bar{a}bc\bar{d}$	u	1	u	0	u
$\bar{a}b\bar{c}d$	u	1	u	1	0
$\bar{a}b\bar{c}\bar{d}$	u	1	u	0	u
$\bar{a}\bar{b}cd$	0	0	u	1	1
$\bar{a}\bar{b}c\bar{d}$	0	0	1	0	u
$\bar{a}\bar{b}\bar{c}d$	0	0	u	1	1
$\bar{a}\bar{b}\bar{c}\bar{d}$	0	0	0	0	u

Table 1. Values for Example 4

So we can derive $(\kappa^Z_{\Delta_0})^{-1}(0) = \{abcd, ab\bar{c}d, a\bar{b}cd, a\bar{b}\bar{c}d, \bar{a}bcd, \bar{a}b\bar{c}d\}$ and $(\kappa^Z_{\Delta_1})^{-1}(0) = \emptyset$.

In some recent works [12] we have already presented these first steps. In these works we first translated an abstract argumentation framework into a conditional logic base, using methods introduced in [8,9]. Using the constructed logic base as well as ranking functions like *System Z* we can then rank the resulting worlds based on their plausibility. So the most plausible world is ranked the highest. Based on this ranking we constructed a way to rank the arguments, which results in an ranking-based semantics [1,5]. The first idea is to count the appearances of each argument in the most plausible worlds. So for *System Z* we count how often a positive literal of an argument is inside $(\kappa^Z_{\Delta_0})^{-1}(0)$.

One approach to evaluate an ranking-based semantics is to check the properties, which this semantics satisfies and which it violates. For that we should first look at the simplest properties like *Void Precedence* [1,10], which says that an argument which is not attack should always be ranked higher then an argument with an attack, or *Self-Contradiction* [10], meaning that an argument which contradict itself should always be ranked worse then any other argument.

This approach for ranking arguments can lead to a new way of reasoning inside formal argumentation and also bridging the gap of plausible conditionals and formal argumentation.

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