On Uncertainty in Collective Attacks in Abstract Argumentation

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Abstract

We introduce *incomplete argumentation frameworks with set attacks* (ISETAFs), a novel generalisation of abstract argumentation frameworks that integrates set-based attacks and incomplete information. This unified model subsumes both *argumentation frameworks with set attacks* (SETAFs) and *incomplete argumentation frameworks* (IAFs) and is designed to better capture uncertainty in real-world argumentative reasoning. We explore two complementary approaches: a completion-based semantics that evaluates all possible completions, and an extension-based semantics that defines new acceptance conditions directly over ISETAFs. We show that key reasoning problems for ISETAFs are no harder than for IAFs, and we analyse which desirable properties (e.g., I-maximality, conflict-sensitivity, directionality) are preserved. Our results demonstrate that ISETAFs maintain the expressiveness and semantic rigour of existing frameworks, while significantly enhancing their modelling capabilities.

Keywords

Abstract Argumentation Frameworks, Incomplete Argumentation Frameworks, Set Argumentation Frameworks

1. Introduction

Formal argumentation [1] is concerned with models of rational decision-making based on representations of arguments and their relations. A particularly important and simple approach is that of abstract argumentation frameworks (AF) [2], which represent argumentative scenarios as directed graphs. Here, *arguments* are identified by vertices, and an *attack* from one argument to another is represented as a directed edge. To reason over AFs *extension semantics* were proposed. These are functions that evaluate whether a set of arguments is accepted or not.

Due to the limited expressivity of abstract argumentation frameworks a number of extensions were proposed. One such extension are *incomplete argumentation frameworks* (IAFs) [3], which can model qualitative uncertainty about the existence of arguments and attacks. Such uncertainty can emerge if there is a disagreement between two agents about the validity of one argument. Another extension are *argumentation frameworks with set attacks* (SETAFs) [4], where an attack is not a binary relation between two arguments, but rather a set of arguments attacks an argument. With this attack notion collective attacks can be modelled, for example the statements "John has hay fever" and "It is hay fever season" are individually not a reason against going on a hike, however together these two statements are a reason against going on a hike.

Both IAFs and SETAFs have higher expressivity than AFs, however these two formalisms cannot model each other. An IAF cannot handle collective attacks and a SETAF cannot cope with uncertainty. In this paper, we combine these two formalisms into *incomplete argumentation frameworks with set attacks* (ISETAFs) to define a more expressive extension of AFs, which can handle both uncertainty and collective attacks. More precisely, the main contributions of this paper are as follows:

- We present *incomplete argumentation frameworks with set attacks* as an extension of abstract argumentation frameworks.
- We discuss two approaches to reason with ISETAFs.

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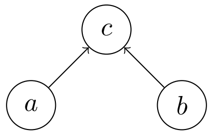

Figure 1: Abstract argumentation framework F_1 from Example 1.

- We evaluate properties for ISETAFs by modifying properties for AFs.
- We analyse the computational complexity of reasoning with ISETAFs.

In Section 2, we recall all necessary background information on abstract argumentation frameworks, incomplete argumentation frameworks, and argumentation frameworks with set attacks. In Section 3, we introduce incomplete argumentation frameworks with set attacks as an extension of AFs. In addition, we discuss two reasoning approaches for this new framework. Properties of extension semantics of ISETAFs are discussed in Section 4. The computational complexity of the corresponding acceptance problems are analysed in Section 5. Section 6 concludes this paper.

Due to space limitations, some proofs can be found in the appendix [1].

2. Preliminaries

Abstract argumentation frameworks [2] are a formalism that allows the representation of conflicts between pieces of information using arguments and attacks between arguments.

Definition 1. *An* abstract argumentation framework *(AF) is a directed graph $F = (A, R)$ where A is a finite set of* arguments *and R is an* attack relation *$R \subseteq A \times A$.*

For an AF $F = (A, R)$, an argument a is said to *attack* an argument b if $(a, b) \in R$. We say that, a set $E \subseteq A$ *defends* an argument a if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$, we define $a_F^- = \{b \mid (b, a) \in R\}$ and $a_F^+ = \{b \mid (a, b) \in R\}$. In other words, a_F^- is the set of attackers of a and a_F^+ is the set of arguments attacked by a. For a set of arguments $E \subseteq A$, we extend these definitions to E_F^+ and E_F^- via $E_F^+ = \bigcup_{a \in E} a_F^+$ and $E_F^- = \bigcup_{a \in E} a_F^-$, respectively. If the AF is clear in the context, we omit the index. To reason with AFs, Dung [2] proposed *extension semantics*.

Definition 2. *Given $F = (A, R)$, a set $E \subseteq A$ is a*

- conflict-free *(*cf*) set iff $\forall a, b \in E$, $(a, b) \notin R$;*
- admissible *(*ad*) set iff it is* conflict-free *and it defends its elements;*
- complete *extension (*co*) iff it contains every argument it defends;*
- preferred *extension (*pr*) iff it is a $\subseteq$-maximal admissible set;*
- grounded *extension (*gr*) iff it is the $\subseteq$-minimal complete extension;*
- stable *extension (*st*) iff E is conflict-free and $E \cup E_F^+ = A$.*

We use $\mathsf{cf}(F)$ and $\mathsf{ad}(F)$ to denote the sets of conflict-free and admissible sets of an argumentation framework F, respectively, and $\mathsf{co}(F)$, $\mathsf{pr}(F)$, $\mathsf{gr}(F)$, and $\mathsf{st}(F)$ to denote the complete, preferred, grounded and stable extensions of F.

Example 1. *Consider the AF $F_1 = (\{a, b, c\}, \{(a, c), (b, c)\})$ depicted as a directed graph in Figure 1. F_1 has four admissible sets $E_1 = \{a\}$, $E_2 = \{b\}$, $E_3 = \{a, b\}$, and $E_4 = \emptyset$. Since arguments a and b are unattacked, only set E_3 is the complete, preferred, grounded, and stable extension.*

An argument a is *credulously accepted* in F with respect to the extension semantics σ if there exists a set $a \in E$ such that $E \in \sigma(F)$, argument a is *skeptically accepted* if $a \in E$ for all $E \in \sigma(F)$.

Incomplete argumentation frameworks [5, 3, 6, 7] are an extension of AFs with qualitative uncertainty about the presence of arguments and attacks.

[1] https://fernuni-hagen.sciebo.de/s/n3wsYP9yIpuUPVa

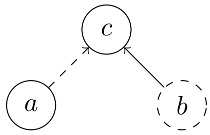

Figure 2: IAF $\mathcal{I}_1$ from Example 2.

Definition 3. *An* incomplete argumentation framework *(IAF) is a tuple* $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$*, where* $\mathcal{A} \cap \mathcal{A}^? = \emptyset$ *and* $\mathcal{R}, \mathcal{R}^?$ *the sets of attacks with* $\mathcal{R}, \mathcal{R}^? \subseteq (\mathcal{A} \cup \mathcal{A}^?) \times (\mathcal{A} \cup \mathcal{A}^?)$.

Elements from $\mathcal{A}$ and $\mathcal{R}$ are certain arguments and attacks (*i.e. the agent is sure that they appear in the framework*), while $\mathcal{A}^?$ and $\mathcal{R}^?$ are uncertain arguments and attacks (*i.e. there is a doubt about their actual existence*). To reason on IAFs we use *completions*. These were first defined for *Partial AFs* (i.e. IAFs with $\mathcal{A}^? = \emptyset$, see [5]) and then adapted to IAFs. Intuitively, a completion is a possible world consistent with the uncertainty encoded in the IAF.

Definition 4. *Given* $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$*, a* completion *of* $\mathcal{I}$ *is an AF* $F = (A', R')$*, such that:*

$$\mathcal{A} \subseteq A' \subseteq \mathcal{A} \cup \mathcal{A}^?;$$
$$\mathcal{R} \cap (A' \times A') \subseteq R' \subseteq (\mathcal{R} \cup \mathcal{R}^?) \cap (A' \times A');$$

The set of completions of an IAF $\mathcal{I}$ is denoted with $\mathsf{comp}(\mathcal{I})$. We also introduce $\mathsf{comp}(\mathcal{I}, a) = \{F = (A', R') \mid F \in \mathsf{comp}(\mathcal{I}) \text{ and } a \in A'\}$ the sets of completions where argument a is present, with $a \in \mathcal{A} \cup \mathcal{A}^?$. Naturally, if $a \in \mathcal{A}$ then $\mathsf{comp}(\mathcal{I}, a) = \mathsf{comp}(\mathcal{I})$.

With these completions, we extend the reasoning process of AFs to IAFs. For an IAF $\mathcal{I}$, an certain or uncertain argument a is *possibly credulously accepted* with respect to an extension semantics σ in $\mathcal{I}$ if there is at least one completion $F \in \mathsf{comp}(\mathcal{I})$ such that a is credulously accepted with respect to σ. Argument a is *necessarily credulously accepted* with respect to σ in $\mathcal{I}$ if a is credulously accepted with respect to σ in every completion of $\mathcal{I}$. *Possible skeptical acceptance* and *necessary skeptical acceptance* are defined accordingly.

Example 2. *Consider the IAF* $\mathcal{I}_1 = (\{a, c\}, \{b\}, \{(b, c)\}, \{(a, c)\})$ *depicted as a directed graph in Figure 2. The attack from* a *to* c *is uncertain and argument* b *is uncertain. This IAF has four completions, one containing* b *and* (a, c)*, one containing only* b *and not* (a, c)*, one containing* (a, c) *and not* b*, and one containing neither* b *nor* (a, c)*. In each of these completions, we check whether an argument is accepted or not according to an extension semantics. For example, argument* a *is necessary skeptically accepted w.r.t. preferred semantics, while arguments* b *and* c *are possible skeptically accepted w.r.t. preferred semantics.*

A second approach to reasoning on IAFs is an *extension-based* approach. Given an IAF, sets of arguments can be evaluated from two perspectives: *optimistic* (weak, w) and *pessimistic* (strong, s), depending on the treatment of uncertainty.

Definition 5. *Let* $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$ *be an IAF,* $E \subseteq \mathcal{A} \cup \mathcal{A}^?$ *and* $x \in \{w, s\}$*. A set* E x-defends *an argument* $a \in \mathcal{A} \cup \mathcal{A}^?$ *if*

- *for* $x = w$*: for every* $b \in \mathcal{A}$ *with* $(b, a) \in \mathcal{R}$*, there exists* $c \in E \cap \mathcal{A}$ *with* $(c, b) \in \mathcal{R}$*;*
- *for* $x = s$*: for every* $b \in \mathcal{A} \cup \mathcal{A}^?$ *with* $(b, a) \in \mathcal{R} \cup \mathcal{R}^?$*, there exists* $c \in E \cap \mathcal{A}$ *with* $(c, b) \in \mathcal{R}$*.*

The semantics are defined as follows:

Definition 6 ([8, 9])**.** *Let* $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$ *be an IAF,* $E \subseteq \mathcal{A} \cup \mathcal{A}^?$*, and* $x \in \{w, s\}$*. Then:*

- E *is* x-conflict-free (cf_x) *if*
  - *for* $x = w$*: for all* $a, b \in E \cap \mathcal{A}$*,* $(a, b) \notin \mathcal{R}$*,*
  - *for* $x = s$*: for all* $a, b \in E$*,* $(a, b) \notin \mathcal{R} \cup \mathcal{R}^?$*.*

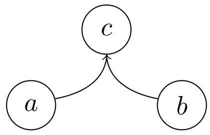

Figure 3: SETAF $\mathtt{S}_1$ from Example 4.

- E *is* x-admissible (ad_x) *iff* E *is* x*-conflict-free and* x*-defends all* $a \in E$.
- E *is* x-complete (co_x) *iff* E *is* x*-admissible and contains all arguments it* x*-defends.*
- E *is* x-preferred (pr_x) *iff* E *is a* $\subseteq$*-maximal* x*-complete set.*
- E *is* x-grounded (gr_x) *iff* E *is the* $\subseteq$*-minimal* x*-complete set.*
- E *is* x-stable (st_x) *iff* E *is* x*-conflict-free and, for all* $a \in A_x \setminus E$, a *is certainly attacked by* E*, where* $A_w = \mathcal{A}$ *and* $A_s = \mathcal{A} \cup \mathcal{A}^?$.

This extension-based approach enables direct evaluation of argument acceptability in IAFs via generalised semantics, allowing for fine-grained reasoning under uncertainty by distinguishing between optimistic and pessimistic interpretations.

Example 3. *Consider again* $\mathcal{I}_1$ *from Example 2. Since the attack from* a *to* c *is uncertain, the set* $\{a, c\}$ *is* w*-admissible. The set* $\{a, b\}$ *is* s*-complete,* s*-preferred, and* s*-grounded.*

Another extension of abstract argumentation frameworks are *argumentation frameworks with set attacks* by Nielsen and Parsons [4], where a set of arguments collectively attack an argument.

Definition 7. *An* argumentation framework with set attacks *(SETAF) is a tuple* $\mathtt{S} = (\mathtt{A}, \mathtt{R})$*, where* $\mathtt{A}$ *is a finite set of arguments and* $\mathtt{R} \subseteq (2^{\mathtt{A}} \setminus \emptyset) \times \mathtt{A}$.

For a SETAF $\mathtt{S} = (\mathtt{A}, \mathtt{R})$, a set of arguments $E \subseteq \mathtt{A}$ attacks argument $a \in \mathtt{A}$ if and only if $(E, a) \in \mathtt{R}$. A set of arguments E defends an argument a if for every $G \subseteq \mathtt{A}$ such that $(G, a) \in \mathtt{R}$ there is an set $E' \subseteq E$ with $(E', b) \in \mathtt{R}$ where $b \in G$. With these attack and defence notions, we can define the extension semantics for SETAF similar to Definition 2.

Example 4. *Consider the SETAF* $\mathtt{S}_1$ *depicted as a directed hypergraph in Figure 3. Arguments* a *and* b *collectively attack argument* c*. Thus, the set* $\{a, b, c\}$ *is not conflict-free, however the two subsets* $\{a, c\}$ *and* $\{b, c\}$ *are conflict-free. The set* $\{a, b\}$ *is the only complete, preferred, grounded, and stable set in* $\mathtt{S}_1$.

3. Incomplete Set Argumentation Framework

Incomplete argumentation frameworks and argumentation frameworks with set attacks both extend the expressivity of argumentation frameworks. Although they extend an AF differently, IAFs can handle uncertainty and SETAFs can handle collective attacks, but they cannot handle the extension of the other. However, an uncertain collective attack is imaginable.

Example 5. *A group of friends want to go on a hike, however John has hay fever, so he has to track the amount of pollen in the air. If there is too much pollen he cannot go hiking.*

If we try to model this scenario with an argumentation framework, we get three arguments "John goes hiking", "John has hay fever", and "the pollen concentration is high". The statements "John has hay fever" and "the pollen concentration is high" only attack "John goes hiking" together, since individually the two statements are no reason against going hiking. We could model this with a SETAF, however, if we consider that we cannot be sure whether the pollen concentration is high or not, we are unsure about the existence of the argument "the pollen concentration is high". This uncertainty cannot be modelled with a SETAF. However, an IAF can model this uncertainty, but it cannot model the collective attack.

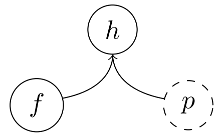

Figure 4: ISETAF U_1 from Example 6.

Example 5 motivates the use of a combination of IAFs and SETAFs to handle uncertain collective attacks. Next, we define *incomplete argumentation frameworks with set attacks* as an extension of IAFs and SETAFs and present the corresponding attack notion.

Definition 8. *An* incomplete argumentation framework with set attacks *(ISETAF) is a tuple* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *where* A *are the certain arguments,* $\mathsf{A}^?$ *uncertain arguments and* $\mathsf{R}, \mathsf{R}^?$ *the sets of collective attacks with* $\mathsf{A} \cap \mathsf{A}^? = \emptyset$ *and* $\mathsf{R}, \mathsf{R}^? \subseteq (2^{\mathsf{A} \cup \mathsf{A}^?} \setminus \emptyset) \times (\mathsf{A} \cup \mathsf{A}^?)$.

For an ISETAF $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$, a set of arguments $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ *attacks* an argument $a \in \mathsf{A} \cup \mathsf{A}^?$ if $(E, a) \in \mathsf{R} \cup \mathsf{R}^?$. An attack (E, a) can consist out of multiple uncertainty. One or multiple arguments of E can be uncertain, the attack relation can be uncertain, or any combination of these two situation can occur. We call an attack (E, a) with $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ and $a \in \mathsf{A} \cup \mathsf{A}^?$ *conditionally certain* if $(E, a) \in \mathsf{R}$ and *certain* if $(E, a) \in \mathsf{R}$ and $E \subseteq \mathsf{A}$. An attack (E, a) is *uncertain* if $(E, a) \in \mathsf{R}^?$ for $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ and $a \in \mathsf{A} \cup \mathsf{A}^?$.

Example 6. *Continuing Example 5, we can model our scenario with an ISETAF as follows:*

$$\mathsf{U}_1 = (\{h, f\}, \{p\}, \{(\{f, p\}, h)\}, \emptyset)$$

The corresponding hypergraph is depicted in Figure 4, where:

- h *represents: John goes hiking*
- f *represents: John has hay fever*
- p *represents: the pollen concentration is high*

The arguments h *and* f *are certain arguments and* p *is uncertain. The set* $\{p, f\}$ *are collectively attacking argument* h*, this attack* $(\{f, p\}, h)$ *is a conditionally certain attack, meaning that if argument* p *exists the attack* $(\{f, p\}, h)$ *exists as well. If argument* p *does not exists, then the attack* $(\{f, p\}, h)$ *has to be removed as well.*

In the remainder of this section, we discuss two reasoning approaches, *completion-based* and *extension-based*, for ISETAFs.

3.1. Completion-based Approach

Similar to incomplete argumentation frameworks, uncertainty in an ISETAF can be resolved with an *completion-based approach.* Each uncertain element is either included in or excluded from the completion, resulting in a SETAF.

Definition 9. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF, a* completion *of* U *is a SETAF* $\mathtt{S} = (\mathtt{A}^*, \mathtt{R}^*)$*, s.t.:*

$$\mathsf{A} \subseteq \mathtt{A}^* \subseteq (\mathsf{A} \cup \mathsf{A}^?)$$
$$\mathsf{R} \cap (2^{\mathtt{A}^*} \times \mathtt{A}^*) \subseteq \mathtt{R}^* \subseteq (\mathsf{R} \cup \mathsf{R}^?) \cap (2^{\mathtt{A}^*} \times \mathtt{A}^*)$$

The set of completions of an ISETAF U is denoted with $\mathsf{comp}_{\mathsf{ISETAF}}(\mathsf{U})$. Using the completions of an ISETAF, we extend the acceptability question to ISETAFs.

Definition 10. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $a \in \mathsf{A} \cup \mathsf{A}^?$ *and* σ *an extension semantics.*

- a *is* possibly credulously accepted *with respect to* σ *in* U *if there is at least one completion of* $\mathsf{S} \in \mathsf{comp}_{\mathsf{ISETAF}}(\mathsf{U})$ *such that* a *is credulously accepted with respect to* σ *in* S.
- a *is* necessarily credulously accepted *with respect to* σ *in* U *if* a *is credulously accepted with respect to* σ *in every completion of* U.
- a *is* possible skeptically accepted *with respect to* σ *in* U *if there is at least one completion of* $\mathsf{S} \in \mathsf{comp}_{\mathsf{ISETAF}}(\mathsf{U})$ *such that* a *is skeptically accepted with respect to* σ *in* S.
- a *is* necessary skeptically accepted *with respect to* σ *in* U *if* a *is skeptically accepted with respect to* σ *in every completion of* U.

The acceptance problems give us insight into the behaviour of the ISETAF. If an argument a is possibly credulously accepted, then there is one interpretation, where a can be accepted. If an argument a is necessary skeptically accepted, then a is always accepted and its acceptance cannot change.

Example 7. *Continuing with Example 6, this ISETAF has two completions, one with* p *and one without* p. *We call the completion with* p $\mathsf{U}_{1,p}$ *and the one without* p $\mathsf{U}_{1,\bar{p}}$. *The set* $\{f, h\}$ *is conflict-free in* $\mathsf{U}_{1,p}$, *while it is not conflict-free in* $\mathsf{U}_{1,\bar{p}}$. *Thus,* h *is possibly credulously accepted with respect to admissibility in* U_1. *Argument* f *is necessary skeptically accepted with respect to complete semantics in* U_1.

To construct a completion for an ISETAF, we have a number of options for a single collective attack. If the attack is certain, then we get a single completion, however if the attack is conditionally certain then we have multiple options. For any uncertain argument that is part of the attack, we can decide to either include it in the completion or leave it out. For uncertain attacks, an additional decision has to be done, to include the collective attack or not, thus the number of completions of an ISETAF scales exponentially to the number of uncertain arguments.

Proposition 1. *Consider ISETAF* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *and* $\mathsf{r}' \in \mathsf{R} \cup \mathsf{R}^?$ *such that* $\mathsf{r}' = (E, a)$ *with* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ *and* $a \in \mathsf{A}$. *Let* $m = |E \cap \mathsf{A}^?|$ *be the number of uncertain arguments of* E *and* $p \in \{0, 1\}$ *such that:*

$$p := \begin{cases} 0 & \text{if } r' \in \mathsf{R} \\ 1 & \text{if } r' \in \mathsf{R}^? \end{cases}$$

Then the number of completions with respect to r' *is* $2^m + p$.

Proof. We prove the proposition by complete induction. Consider a single collective attack of an ISETAF. Let m denote the number of uncertain arguments involved, and let $p \in \{0, 1\}$ indicate whether the attack itself is uncertain ($p = 1$) or certain ($p = 0$). We show that the number of completions is $n = 2^m + p$.

- **Base case** ($m = 0$): If no uncertain arguments are involved, the number of completions depends only on whether the attack is certain or uncertain.
  - If the attack is certain ($p = 0$): $n = 2^0 + 0 = 1$.
  - If the attack is uncertain ($p = 1$): $n = 2^0 + 1 = 2$.

  Thus, the base case holds.
- **Induction hypothesis:** Assume the formula holds for k uncertain arguments, i.e., $n = 2^k + p$.
- **Induction step:** Consider $m = k + 1$, i.e., one additional uncertain argument.
  - If $p = 0$: Adding another uncertain argument doubles the number of completions, as each previous completion can either include or exclude the new uncertain argument. Thus, $n = 2 \cdot 2^k = 2^{k+1}$.
  - If $p = 1$: For k uncertain arguments, we have $2^k + 1$ completions. Adding one more uncertain argument doubles the completions not including the attack and leaves the single completion with the attack, so $n = 2 \cdot 2^k + 1 = 2^{k+1} + 1$.

  In both cases, the formula $n = 2^m + p$ holds for $m = k + 1$. □

The total number of completions for an ISETAF can be determined by multiplying the number of completions for each attack, provided that all collective attacks are independent, i.e., no argument occurs in more than one collective attack. Thus, the number of completions of an SETAF grow exponentially.

3.2. Extension-based Approach

To avoid the exponential amount of completions for an IAF, Mailly proposed an *extension-based approach* to reason with IAFs [8, 9, 10]. To handle uncertainty there are two point of views, *optimistic* and *pessimistic*. In the optimistic view uncertain arguments and attacks are considered not harmful, thus conflicts originating from uncertain attacks are tolerated. In the pessimistic view all attacks are harmful. We adapt these two view points for ISETAFs.

Definition 11. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF and* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$. *The set* E *is:*

- weakly conflict-free *iff there is no attack* $(E', a) \in \mathsf{R}$ *with* $E' \subseteq E \cap \mathsf{A}$ *and* $a \in E$.
- strongly conflict-free *iff there is no attack* $(E', a) \in \mathsf{R} \cup \mathsf{R}^?$ *with* $E' \subseteq E \cap (\mathsf{A} \cup \mathsf{A}^?)$ *and* $a \in E$.

With $\mathsf{cf}_w(\mathsf{U})$ and $\mathsf{cf}_s(\mathsf{U})$ we denote the sets of weakly respectively strongly conflict-free sets of an ISETAF U. The second fundamental concept of formal argumentation is *defence*.

Definition 12. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ *and* $a \in \mathsf{A} \cup \mathsf{A}^?$. *The set* E*:*

- weakly defends a *iff for every certain collective attack* $(E', a) \in \mathsf{R}$ *with* $E' \subseteq \mathsf{A}$*, there exists a nonempty subset* $G \subseteq E \cap \mathsf{A}$ *such that* $(G, b) \in \mathsf{R}$ *for at least one* $b \in E'$.
- strongly defends a *iff for every collective attack* $(E', a) \in \mathsf{R} \cup \mathsf{R}^?$ *with* $E' \subseteq \mathsf{A} \cup \mathsf{A}^?$*, there exists a nonempty subset* $G \subseteq E \cap \mathsf{A}$ *such that* $(G, b) \in \mathsf{R}$ *for at least one* $b \in E'$.

Next, we combine both conflict-freeness and defence to extension semantics for ISETAFs similar to Definition 2. For every extension semantics, we get two variations, a *weak* and a *strong* one.

Definition 13. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ *and* $x \in \{w, s\}$. *Then:*

- E *is* x-admissible *(*ad_x*) iff* E *is* cf_x *and* E x*-defends all its elements;*
- E *is* x-complete *(*co_x*) iff* E *is* ad_x *and every* $a \in \mathsf{A} \cup \mathsf{A}^?$ x*-defended by* E *is in* E*;*
- E *is* x-preferred *(*pr_x*) iff* E *is a* $\subseteq$*-maximal* x*-complete set;*
- E *is* x-grounded *(*gr_x*) iff* E *is the* $\subseteq$*-minimal* x*-complete set;*
- E *is* x-stable *(*st_x*) iff* E *is* cf_x *and for all* $a \in A_x \setminus E$*,* a *is certainly attacked by* E*, where* $A_w = \mathsf{A}$*,* $A_s = \mathsf{A} \cup \mathsf{A}^?$.

Example 8. *We modify the ISETAF from Example 6 to* $\mathsf{U}_2 = (\{h, f\}, \{p\}, \emptyset, \{(\{f, p\}, h)\})$*, thus the collective attack is uncertain. Since argument* h *is only attacked by an uncertain collective attack, the set* $\{f, h, p\}$ *is* w*-complete, while the set* $\{f, p\}$ *is* s*-complete.*

4. Properties

In this section, we investigate key properties for the extension-based approach for ISETAF, thus we focus on the ISETAF directly and avoid generating completions. The properties considered were originally introduced for standard AF [11], but are extended to fit the context of ISETAFs with minor modifications.

We begin by discussing the relationship between ISETAFs, AFs, IAFs, and SETAFs. By construction, ISETAFs subsumes AFs, IAFs, and SETAFs as special cases: for AFs, all attacks are certain and every set-attack is a singleton; for IAFs, all set-attacks are singletons (which may be certain or uncertain); and for SETAFs, all set-attacks are certain (but may have arbitrary size). As a result, properties and results established for the classical frameworks are directly inherited or can be generalised for ISETAFs. To make this correspondence precise, we formally state and prove the following proposition:

Proposition 2. *Every AF, IAF, and SETAF can be represented as an ISETAF.*

Proof. Let $F = (A, R)$ be an AF with $R \subseteq A \times A$. The corresponding ISETAF is $\mathsf{U} = (A, \emptyset, \{(\{a\}, b) \mid (a, b) \in R\}, \emptyset)$. Thus, for any AF, a corresponding ISETAF can be constructed.

Let $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$ be an IAF. The corresponding ISETAF is $\mathsf{U} = (\mathcal{A}, \mathcal{A}^?, \{(\{a\}, b) \mid (a, b) \in \mathcal{R}\}, \{(\{a\}, b) \mid (a, b) \in \mathcal{R}^?\})$. Thus, for any IAF, a corresponding ISETAF can be constructed.

Let $\mathtt{S} = (\mathtt{A}, \mathtt{R})$ be a SETAF. The corresponding ISETAF is $\mathsf{U} = (\mathtt{A}, \emptyset, \mathtt{R}, \emptyset)$. Thus, for any SETAF, a corresponding ISETAF can be constructed. □

Properties originally considered for AFs can be meaningfully adapted to ISETAFs, taking into account the richer structure provided by uncertain arguments and collective attacks. For each property, we carefully reformulated the postulate in the ISETAF setting and examined its satisfaction for all extension-based semantics, distinguishing between their weak and strong variants. A comprehensive overview of all results is presented in tabular form at the end of the section.

To analyse the properties within ISETAFs, several technical notions must first be formally introduced. In particular, we require precise definitions of *set of attack arguments*, *x-untouched sets*, *projection operator*, *cohesive subsets*, and the *x-reduct* of an ISETAF. These concepts provide the formal basis for our property formulations and subsequent results. We proceed by outlining the required definitions before presenting and discussing the adapted properties in detail.

Definition 14. *Let $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ be an ISETAF and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ a set of arguments. We define:*

- $E^{+,w} := \{a \mid (E', a) \in \mathsf{R}$ *s.t.* $E' \subseteq E\}$ *– the set of arguments certainly attacked by E;*
- $E^{+,s} := \{\, a \mid (E', a) \in \mathsf{R} \cup \mathsf{R}^?$ *s.t.* $E' \subseteq E\}$ *– the set of arguments certainly or uncertainly attacked by E.*

A set of arguments E is considered *untouched* if it is not attacked by any set of arguments.

Definition 15. *Let $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ be an ISETAF and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ a set of arguments. E is:*

- w-untouched *if there is no $b \in \mathsf{A} \setminus E$ and no set $B \subseteq \mathsf{A}$ with $b \in B$ such that $(B, a) \in \mathsf{R}$ for any $a \in E$.*
- s-untouched *if there is no $b \in (\mathsf{A} \cup \mathsf{A}^?) \setminus E$ and no set $B \subseteq \mathsf{A} \cup \mathsf{A}^?$ with $b \in B$ such that $(B, a) \in \mathsf{R} \cup \mathsf{R}^?$ for any $a \in E$.*

A *projection* of an ISETAF with respect to a set of arguments E is an ISETAF, where every argument not in E is removed.

Definition 16. *Let $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ be an ISETAF and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$. The* projection *of U onto E, denoted $\mathsf{U}_{\downarrow E}$, is $\mathsf{U}_{\downarrow E} = (\mathsf{A} \cap E,\ \mathsf{A}^? \cap E,\ \mathsf{R}_{\downarrow E},\ \mathsf{R}^?_{\downarrow E})$, where $\mathsf{R}_{\downarrow E} = \{(B, a) \in \mathsf{R} \mid B \subseteq E,\ a \in E\}$ and $\mathsf{R}^?_{\downarrow E} = \{(B, a) \in \mathsf{R}^? \mid B \subseteq E,\ a \in E\}$.*

We call a set *cohesive* if it represents exactly those arguments that collectively participate in a set-based attack. More formally, a singleton $\{a\}$ is cohesive if the argument a alone forms a set attack, i.e., there exists a target c such that $(\{a\}, c)$ is a set attack. In contrast, if a does not attack any argument on its own, then $\{a\}$ is not cohesive. Furthermore, if a group of arguments, such as a, b, and c, jointly attack a common target, then the set $\{a, b, c\}$ is cohesive.

Definition 17. *Let $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ be an ISETAF. A set $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ is called a* cohesive set *if there exists $c \in \mathsf{A} \cup \mathsf{A}^?$ such that $(E, c) \in \mathsf{R} \cup \mathsf{R}^?$.*

The *reduct* of an ISETAF U w.r.t. a set E is an ISETAF, where all arguments of E and arguments attacked by E are removed. All attacks involving the removed arguments are removed as well.

Definition 18. *Let $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ be an ISETAF and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$. The weak or strong x-*reduct *of U with respect to E, denoted $\mathsf{U}^{E,x} = (\mathsf{A}', \mathsf{A}'^?, \mathsf{R}', \mathsf{R}'^?)$, is defined by:*

$$\mathsf{A}' = \mathsf{A} \setminus (E \cup E^{+,x}),$$

$$\mathsf{A}'^? = \mathsf{A}^? \setminus (E \cup E^{+,x}),$$
$$\mathsf{R}' = \{(B \setminus E,\, a) \mid (B,a) \in \mathsf{R},\, B \subseteq \mathsf{A}' \cup \mathsf{A}'^?,\, a \in \mathsf{A}' \cup \mathsf{A}'^?,\, B \cap E^{+,x} = \emptyset\},$$
$$\mathsf{R}'^? = \{(B \setminus E,\, a) \mid (B,a) \in \mathsf{R}^?,\, B \subseteq \mathsf{A}' \cup \mathsf{A}'^?,\, a \in \mathsf{A}' \cup \mathsf{A}'^?,\, B \cap E^{+,x} = \emptyset\}.$$

With the necessary background cleared, we can define properties for ISETAFs.

Definition 19. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF, let* σ_x *be an extension-based semantics* ($x \in \{w, s\}$)*, and* $\sigma_x(\mathsf{U})$ *the set of corresponding extensions.*

We say that σ_x *satisfies the following property if and only if:*

- ***Language independence:*** *For any two ISETAFs* U, U' *and any bijective renaming* ρ *such that* $\mathsf{U}' = \rho(\mathsf{U})$ *and* ρ *preserves the type of each argument (certain or uncertain) and each attack (certain or uncertain), it holds that* $\sigma_x(\mathsf{U}') = \rho(\sigma_x(\mathsf{U}))$.
- ***I-maximality:*** *For all* $E, E' \in \sigma_x(\mathsf{U})$*, if* $E \subseteq E'$*, then* $E = E'$*, i. e., no two extensions are in a proper subset relation.*
- ***Allowing abstention:*** *For all* $E, E' \in \sigma_x(\mathsf{U})$ *and all* $a \in \mathsf{A} \cup \mathsf{A}^?$*: if* $a \in E$ *and* $a \in E'^{+,x}$*, then there exists* $E'' \in \sigma_x(\mathsf{U})$ *such that* $a \notin E''$ *and* $a \notin E''^{+,x}$.
- ***Set-based directionality:*** *For every* x*-untouched set* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$ *in* U*, it holds that* $\sigma_x(\mathsf{U}_{\downarrow E}) = \{E' \cap E \mid E' \in \sigma_x(\mathsf{U})\}$.
- ***Set-based tightness:*** *For any* $E \in \sigma_x(\mathsf{U})$ *and any* $a \in (\mathsf{A} \cup \mathsf{A}^?) \setminus E$ *with* $E \cup \{a\} \notin \sigma_x(\mathsf{U})$*, there exists a cohesive set* $B \subseteq E$ *such that for all* $E' \in \sigma_x(\mathsf{U})$*,* $B \cup \{a\} \not\subseteq E'$.
- ***Set-based conflict sensitivity:*** *For all* $E, E' \in \sigma_x(\mathsf{U})$ *with* $E \cup E' \notin \sigma_x(\mathsf{U})$*, there exist cohesive sets* $B \subseteq E$ *and* $B' \subseteq E'$ *such that there is no extension* $E'' \in \sigma_x(\mathsf{U})$ *with* $B \cup B' \subseteq E''$.
- ***Modularisation:*** *For any* $E \in \sigma_x(\mathsf{U})$ *and* $E' \in \sigma_x(\mathsf{U}^{E,x})$*, it holds that* $E \cup E' \in \sigma_x(\mathsf{U})$.

The properties Language independence, I-maximality, Allowing abstention, and Modularisation are adapted from AFs to ISETAFs to ensure that the extensions-based semantics for ISETAFs behave inline with their corresponding AF variant. The properties Set-based directionality, Set-based tightness, and Set-based conflict sensitivity help to design algorithms suited for ISETAFs.

Proposition 2 shows that ISETAFs are generalisations of AFs, IAFs, and SETAFs, thus we can immediately observe that any negative result for a given semantics in the classical frameworks will also hold in ISETAF semantics.

Proposition 3. *If an extension semantics* σ *for AFs, SETAFs, or IAFs violates a property* P*, then neither the ISETAF semantics* σ_w *nor* σ_s *satisfies this property* P.

Proof. Let F be an AF, S a SETAF, and $\mathcal{I}$ an IAF. Consider extension semantics σ such that σ violates property P for F, S, or $\mathcal{I}$. Each of these frameworks can be transformed into an ISETAF as seen in Proposition 2, thus σ also violates P in the corresponding ISETAF, implying that this property cannot generally be satisfied. □

Example 9. *The complete semantics violates* I-maximality *for AFs and therefore this property is also violated by the weak and strong complete semantics* (co_w, co_s) *for ISETAFs. Similarly, the property* directionality *is violated by the stable semantics* st *in AFs, and also by* st_w *and* st_s *in ISETAFs.*

The results of our analysis are summarised in Table 1. Each row corresponds to one of the adapted properties, while columns indicate for each ISETAF semantics (both weak and strong variants) whether the property is satisfied (✓) or not (✗).

Proposition 4. *The satisfied and violated by* σ_x *for* $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{pr}, \mathsf{gr}, \mathsf{st}\}$ *and* $x \in \{w, s\}$ *are depicted in Table 1.*

	cf_w	cf_s	ad_w	ad_s	co_w	co_s	pr_w	pr_s	gr_w	gr_s	st_w	st_s
Language independence	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I-maximality	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓	✗	✓
Allowing abstention	✓	✓	✓	✓	✗	✓	✗	✗	✓	✓	✗	✗
Set-based directionality	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗
Set-based tightness	✓	✓	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓
Set-based conflict sensitivity	✓	✓	✓	✓	✗	✗	✓	✓	✓	✓	✓	✓
Modularization	✗	✗	✓	✗	✓	✗	✓	✗	✓	✗	✓	✓

Table 1
Overview of property satisfaction for ISETAF-semantics.

The results demonstrate that ISETAFs inherit many desirable properties of classical argumentation frameworks when appropriately generalised. Most notably, properties such as *language independence, I-maximality, allowing abstention*, and *modularisation* are still satisfied by the considered semantics. In addition, the new properties specifically tailored to ISETAFs, such as *set-based directionality, set-based tightness*, and *set-based conflict sensitivity*, are also satisfied by the respective semantics whenever their analogues hold in the classical AF. Overall, the results show that ISETAFs maintain strong theoretical foundations while providing increased expressiveness for modelling argumentation under incomplete information.

5. Computational Complexity

We analysed ISETAFs based on their axiomatic properties, however beyond these principles, we can also investigate the computational complexity of reasoning with ISETAFs. In this paper, we focus on *decision problems*, i. e. problems consisting out of an instance and a question that has a *YES* or *NO* answer. We assume familiarity with the basic concepts of computational complexity, in particular the complexity classes P, NP, and coNP [12, 13] as well as Π_2^{P} and Π_3^{P}, which are problems that can be recognised by a non-deterministic polynomial time algorithm with access to an NP respectively Π_2^{P} oracle [14].

For abstract argumentation three decision problems are typically considered: *verification*, *credulous acceptance* and *skeptical acceptance* [15, 16]. The verification problem $\textsc{Ver}_\sigma$ asks whether a set E is a σ-extension of an AF or not. Formally:

$\textsc{Ver}_\sigma$	**Input:**	An AF $F = (A, R)$ and $E \subseteq A$
	Output:	`Yes` if $E \in \sigma(F)$, and No otherwise.

The two acceptance problems ask whether a given argument is credulously or skeptically accepted with respect to an extension semantics σ in an AF.

$\textsc{Cred}_\sigma$	**Input:**	An AF $F = (A, R)$ and $a \in A$
	Output:	`Yes` if a is credulously accepted with respect to σ in F, and No otherwise.

$\textsc{Skep}_\sigma$	**Input:**	An AF $F = (A, R)$ and $a \in A$
	Output:	`Yes` if a is skeptically accepted with respect to σ in F, and No otherwise.

The three decision problems can be extended for IAFs to incorporate the *possible* and *necessary* variations of these problems. For instance the possible verification problem p-$\textsc{Ver}_\sigma$ asks whether a set E is a σ-extension of at least one completion of an IAF.

p-$\textsc{Ver}_\sigma$	**Input:**	An IAF $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$ and $E \subseteq \mathcal{A} \cup \mathcal{A}^?$
	Output:	`Yes` if there is a completion $F \in \mathsf{comp}(\mathcal{I})$ such that $E \in \sigma(F)$, and No otherwise.

For the necessary variation of the verification problem a set E has to be a σ-extension in every completion.

N-VER$_\sigma$ **Input:** An IAF $\mathcal{I} = (\mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^?)$ and $E \subseteq \mathcal{A} \cup \mathcal{A}^?$
Output: Yes if for all completions $F \in \mathsf{comp}(\mathcal{I})$ it holds that $E \in \sigma(F)$, and No otherwise.

The possible and necessary variants of the skeptical acceptance problems are defined accordingly [6].

We can extend the six problems (P-VER$_\sigma$, N-VER$_\sigma$, P-CRED$_\sigma$, N-CRED$_\sigma$, P-SKEP$_\sigma$, and N-SKEP$_\sigma$) to ISETAFs. Formally:

P-SETVER$_\sigma$ **Input:** An ISETAF $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$
Output: Yes if there is a completion $\mathtt{S} \in \mathsf{comp}_{\mathsf{ISETAF}}(\mathsf{U})$ such that $E \in \sigma(\mathtt{S})$, and No otherwise.

N-SETVER$_\sigma$ **Input:** An ISETAF $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ and $E \subseteq \mathsf{A} \cup \mathsf{A}^?$
Output: Yes if for all completions $\mathtt{S} \in \mathsf{comp}_{\mathsf{ISETAF}}(\mathsf{U})$ it holds that $E \in \sigma(\mathtt{S})$, and No otherwise.

The acceptance problems (P-SETCRED$_\sigma$, N-SETCRED$_\sigma$, P-SETSKEP$_\sigma$, and N-SETSKEP$_\sigma$) can be defined accordingly. In the following, we analyse the computational complexity of these six decision problems.

Every IAF can be represented as an ISETAF, as shown in Proposition 2. Therefore, reasoning problems for ISETAFs are at least as hard as for IAFs. To establish whether the converse also holds, that is, whether the complexity of ISETAFs is upper-bounded by that of IAFs, it suffices to show that any ISETAF can be translated into an equivalent IAF in polynomial time, while preserving semantic properties.

Inspired by Modgil and Bench-Capon [17], the idea is to simulate each set attack by introducing auxiliary arguments and appropriate attack relations, eliminating collective attacks in favour of standard ones. The following definition formalises this transformation.

Definition 20. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF. The* transformed IAF $\mathcal{I}_\mathsf{U} = (\mathcal{A}_\mathsf{U}, \mathcal{A}^?_\mathsf{U}, \mathcal{R}_\mathsf{U}, \mathcal{R}^?_\mathsf{U})$ *is defined as follows:*

- $\mathcal{A}_\mathsf{U} = \mathsf{A} \cup \{h_a \mid a \in E,\ (E,b) \in \mathsf{R} \cup \mathsf{R}^?\} \cup \{h_{(E,b)} \mid (E,b) \in \mathsf{R} \cup \mathsf{R}^?\}$
- $\mathcal{A}^?_\mathsf{U} = \mathsf{A}^?$
- $\mathcal{R}_\mathsf{U} = \{(a, h_a) \mid a \in E,\ (E,b) \in \mathsf{R} \cup \mathsf{R}^?\} \cup \{(h_a, h_{(E,b)}) \mid a \in E,\ (E,b) \in \mathsf{R} \cup \mathsf{R}^?\} \cup \{(h_{(E,b)}, b) \mid (E,b) \in \mathsf{R}\}$
- $\mathcal{R}^?_\mathsf{U} = \{(h_{(E,b)}, b) \mid (E,b) \in \mathsf{R}^?\}$

This transformation introduces two types of auxiliary arguments: for each attacker in a set attack, and for each set attack as a whole. As a result, all set attacks are simulated via standard attacks between these arguments. To show that the transformation induces the same σ-extensions, we need a few more notations. Besides transforming the ISETAF into an IAF, we also need to transform the corresponding completions.

Definition 21. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $\mathcal{I}_\mathsf{U} = (\mathcal{A}_\mathsf{U}, \mathcal{A}^?_\mathsf{U}, \mathcal{R}_\mathsf{U}, \mathcal{R}^?_\mathsf{U})$ *the transformed IAF with respect to* U *and* $\mathtt{U}^* = (\mathtt{A}^*, \mathtt{R}^*)$ *a completion of* U *with* $\mathsf{A} \subseteq \mathtt{A}^* \subseteq (\mathsf{A} \cup \mathsf{A}^?)$ *and* $\mathsf{R} \cap (2^{\mathtt{A}^*} \times \mathtt{A}^*) \subseteq \mathtt{R}^* \subseteq (\mathsf{R} \cup \mathsf{R}^?) \cap (2^{\mathtt{A}^*} \times \mathtt{A}^*)$. *The corresponding* transformed completion *of* $\mathtt{U}^*$ *is:*

$$I^*_{\to\mathtt{U}^*} = (B, T)$$
$$B = \{\mathcal{A}_\mathsf{U} \cup (\mathcal{A}^?_\mathsf{U} \cap \mathtt{A}^*)\}$$
$$T = \{(c,d) \mid c,d \in B, (c,d) \in \mathcal{R}_\mathsf{U}\} \cup \{(h_{(E,a)}, a) \mid E \subseteq B, (E,a) \in \mathcal{R}^*\}$$

The transformed completion contains all auxiliary arguments, contained in the completion, and therefore also all attacks. Furthermore, for all collective attacks (E, a) of the completions of the ISETAF there is an corresponding attack $(h_{(E,a)}, a)$, from the auxiliary argument to argument a.

Next, we introduce the notion of *transformed sets.*

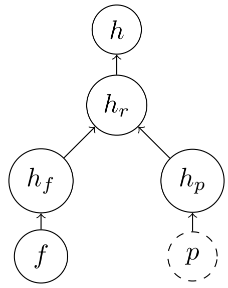

Figure 5: Transformed IAF $\mathcal{I}_{\mathsf{U}_1}$ from Example 10, where $r = (\{f, p\}, h)$.

Definition 22. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $\mathcal{I}_\mathsf{U} = (\mathcal{A}_\mathsf{U}, \mathcal{A}^?_\mathsf{U}, \mathcal{R}_\mathsf{U}, \mathcal{R}^?_\mathsf{U})$ *the transformed IAF with respect to* U*,* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$*,* $\mathtt{U}^* = (\mathtt{A}^*, \mathtt{R}^*)$ *a completion of* U*, and* $I^*_{\to \mathsf{U}^*} = (B, T)$ *the corresponding transformed completion of* U^**.*

The transformed set *with respect to* E *is:*

$$E'_{\to E} = E \cup H_E \cup H'_E \qquad \textit{where:} \qquad \begin{aligned} H_E &= \{h_{(C,a)} \mid C \subseteq E\} \\ H'_E &= \{h_a \mid h_{(C,a)} \in H_E \textit{ or } a \in \mathcal{A}^?_\mathsf{U}, a \notin B\} \end{aligned}$$

Example 10. *Consider* U_1 *from Example 6. For the collective attack* $(\{f, p\}, h)$*, we add three auxiliary arguments* h_f*,* h_p*, and* $h_{(\{f,p\},h)}$*.* h_f *and* h_p *are attacking* $h_{(\{f,p\},h)}$ *and are attacked by* f *and* p*, respectively.* $h_{(\{f,p\},h)}$ *attacks* h*. The corresponding IAF is depicted in Figure 5.*

The set $\{f, p\}$ *is a complete extension in the completion where* p *is contained. The corresponding transformed set is* $\{f, p, h_{(\{f,p\},h)}\}$*, this set is a complete extension. Thus, we find for a complete extension in the ISETAF, a corresponding complete extension in the IAF.*

Example 10 demonstrates, that for every σ-extension of an ISETAF, we can find a corresponding σ-extension in the transformed IAF.

Proposition 5. *Let* $\mathsf{U} = (\mathsf{A}, \mathsf{A}^?, \mathsf{R}, \mathsf{R}^?)$ *be an ISETAF,* $\mathcal{I}_\mathsf{U} = (\mathcal{A}_\mathsf{U}, \mathcal{A}^?_\mathsf{U}, \mathcal{R}_\mathsf{U}, \mathcal{R}^?_\mathsf{U})$ *the transformed IAF with respect to* U*,* σ *an extension semantics,* $E \subseteq \mathsf{A} \cup \mathsf{A}^?$*, and* $E'_{\to E}$ *the transformed set wrt.* E*. Then:*

1. $E'_{\to E}$ *is a possible* σ*-extension in* $\mathcal{I}_\mathsf{U}$ *iff* E *is a possible* σ*-extension in* U*.*
2. $E'_{\to E}$ *is a necessary* σ*-extension in* $\mathcal{I}_\mathsf{U}$ *iff* E *is a necessary* σ*-extension in* U*.*

We can transform any IAF into an ISETAF and back, while inducing the same extensions, thus the computational complexity of p-setVer$_\sigma$, n-setVer$_\sigma$, p-setCred$_\sigma$, n-setCred$_\sigma$, p-setSkep$_\sigma$, and n-setSkep$_\sigma$ coincide with their IAFs counterparts. Table 2 summarises the results.

	p-setCred$_\sigma$	n-setCred$_\sigma$	p-setSkep$_\sigma$	n-setSkep$_\sigma$	p-setVer$_\sigma$	n-setVer$_\sigma$
cf	in P	in P	trivial	trivial	in P	in P
ad	NP-c	Π_2^P-c	trivial	trivial	in P	in P
co	NP-c	Π_2^P-c	NP-c	coNP-c	in P	in P
pr	NP-c	Π_2^P-c	Π_3^P-c	Π_2^P-c	Π_2^P-c	coNP-c
gr	NP-c	coNP-c	NP-c	coNP-c	in P	in P
st	NP-c	Π_2^P-c	Π_2^P-c	coNP-c	in P	in P

Table 2
The complexity of reasoning problems for ISETAFs is derived from the complexity results for IAFs as established in the works by Baumeister et al., and Baumeister et al. [3, 18].

Proposition 6. *The computation complexity of* p-setVer$_\sigma$*,* n-setVer$_\sigma$*,* p-setCred$_\sigma$*,* n-setCred$_\sigma$*,* p-setSkep$_\sigma$*, and* n-setSkep$_\sigma$ *for* $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{pr}, \mathsf{gr}, \mathsf{st}\}$ *are as depicted in Table 2.*

6. Conclusion

We introduced ISETAFs, a novel extension of abstract argumentation frameworks that unifies reasoning with collective attacks and incomplete information within a single model addressing realistic scenarios previously not supported by existing frameworks.

We formally defined ISETAFs and developed two reasoning approaches: a completion-based approach, which evaluates all possible scenarios, and an extension-based approach, which enables direct analysis of argument sets without generating completions. For the extension-based method, we devised weak and strong variants of standard Dung semantics, reflecting different attitudes towards uncertain information.

In addition, we looked at the computational complexity of the corresponding acceptance problems. Our complexity analysis showed that reasoning in ISETAFs does not increase the computational complexity compared to established frameworks, since ISETAF problems can be polynomially reduced to IAFs. At the same time, ISETAFs strictly generalise AFs, SETAFs, and IAFs, preserving their expressiveness but also inheriting both positive and negative results regarding desirable semantic properties.

A systematic analysis of central properties demonstrated that satisfaction of these properties in ISETAFs precisely mirrors their satisfaction in the respective base frameworks and semantics.

Overall, ISETAFs provide a powerful and unifying framework for argumentation under both collective attacks and incomplete information. They enable more expressive and realistic modelling of decision-making scenarios while maintaining manageable reasoning complexity for standard tasks.

Dimopoulos et al. [19] introduced a framework also termed ISETAF, which allows attacks from sets of arguments to sets of arguments and models uncertainty exclusively on attacks, while arguments remain strictly certain. In contrast, our notion of ISETAFs permits uncertainty for both arguments and attacks, and restricts collective attacks to target single arguments. This distinction highlights two different approaches to extending classical argumentation frameworks towards handling incomplete information. In the literature we can find more works discussing the combination of uncertainty and other features. For instance Lagasquie-Schiex et al. [20] discuss the combination of support and uncertainty and Doutre et al. [21] investigates the combination of higher-order attacks and uncertainty. Uncertainty has not only been discussed in the abstract case, but also for structured approaches. Odekerken et al. [22] investigated uncertainty for ASPIC$^+$.

Future work could focus on incorporating probabilistic information into the modelling of uncertain arguments and attacks, following ideas from probabilistic argumentation frameworks [23]. Furthermore, alternative semantics and additional desirable properties, as recently explored for SETAFs [24], could be investigated within the context of incomplete information. Finally, practical implementations and evaluations in real-world settings would help to demonstrate the applicability and potential of the proposed framework.

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